Calculus 140, section 4.6 Extreme Values on an Arbitrary Interval

notes by Tim Pilachowski

Oh boy—now we get to start on word problems!

Example A: The function $s(t) = -9.6t^2 + 48t + 600$ calculates the height of an object, *s*, after time, *t*, thrown upward at 48 meters per second from a bridge which is 600 meters above the river below.

a) What is the domain for this situation?

b) How long does it take for the rock to reach its maximum height? *Answer*: 2.5 sec (How can we verify that this is a maximum?)

c) What is the maximum height above the water reached by the rock? *Answer*: 660 m

d) What is the maximum height above the bridge reached by the rock? *Answer*: 60 m

Example B: Public health officials use rates of change to quantify the spread of an epidemic into an equation, which they then use to determine the most effective measures to counter it. A recent measles epidemic followed the equation $y = 45t^2 - t^3$ where *y* = the number of people infected and *t* = time in days.

a) What is the domain of this function? *Answer*: $0 < t < 45$ days

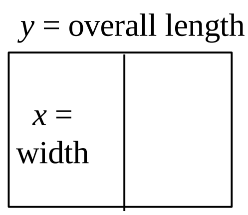
b) What is the rate of spread after 5 days? *Answer*: 375 new cases per day

c) After how many days does the number of cases reach its maximum? *Answer*: 30 days

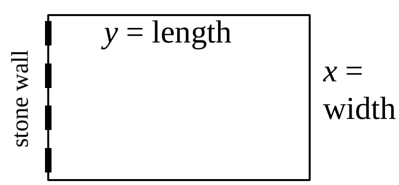
Example C: An efficiency study of the 8 am to noon shift at a factory shows that the number of units, *N*, produced by an average worker *t* hours after 8 am is modeled by the function $N(t) = -t^3 + 9t^2 + 12t$. At what time is the number of units produced at its peak? *Answer*: noon

Example D: Determine two positive numbers whose sum is 20, and whose product is as large as possible.
Answer: $x = y = 10$

Example E: Perhaps you have already encountered versions of the infamous corral problem favored by Math 113 and 220 course coordinators. A farmer has 900 feet of fencing with which to build a pen for his animals, and being a frugal sort doesn't want to buy any more fencing. He needs two pens, but can build them adjacent to each other, sharing one side as in the diagram to the left. Find the dimensions that will give the maximum area. *Answer*: 225 ft long by 150 ft wide



Example F: Farmer Al needs to fence in 800 yd^2, with one wall being made of stone which costs \$24 per yard, and the other three sides being wire mesh which costs \$8 per yard. What dimensions will minimize cost?
Answer: 40 yd long by 20 yd wide



Example G: Optimization does not always involve a maximum. The fuel, maintenance and labor costs (in dollars per mile) of operating a truck on an interstate highway are described as a function of the truck's velocity (miles per hour) by the algebraic rule $C(v)=78+1.2v+5880v^{-1}$. What speed should the driver maintain on a 600 mile haul to minimize costs? *Answer*: 70 mph.

Example H: Determine the area of the largest rectangle that can fit inside a semicircle of radius 1. *Answer*: 1

Example I: What point on the unit circle is nearest to the point (3, –4)? *Answer*: $\left(\frac{3}{5}, -\frac{4}{5}\right)$

Example J: The concentration of a drug in the bloodstream t hours after injection into a muscle is given by $c(t)=9\left(e^{-0.3t}-e^{-3t}\right)$ units. Find the time at which the concentration of the drug in the bloodstream is at its maximum. *Answer*: $\frac{10}{27}\ln 10$ hours